CAPACITANCE

* Functions of capacitors in simple circuits :

→ used to store electrical charge & energy.

→ used in computers: charged up in normal use & then gradually discharge if there is a power failure so that the comp. will operate long enough to save valuable data.

→ also provides extra energy when needed such as using the flash in a camera – the capacitor connected to the bulb discharges rapidly to give a short but intense flash.

→ a simple capacitor consists of 2 leads connected to 2 metal plates (where the charge is stored) with an air gap between them which acts as an insulator. ⇒ "parallel-plate capacitor"

→ the capacitance of an air-filled capacitor can be ↑ by putting an insulating material (mica / waxed paper) between the plates ⇒ "dielectric".

→ in a type of capacitor called ~~eletr~~ electrolytic capacitor, the dielectric is deposited by an electrochemical reaction.

→ These capacitors must be connected with the correct polarity for their plates or they will be ~~g~~ damaged.

→ Capacitor :  Electrolytic capacitor : 

* The capacitance of a capacitor is the charge stored per unit of potential difference across it.

$$\text{Capacitance } (C) = \frac{\text{Charge } (Q)}{\text{p.d. } (V)}$$

* Capacitance is the ratio of charge to potential for a conductor.
* Unit: Farad (F) = one coulomb per volt. ($F = CV^{-1}$)

* Energy stored in a capacitor:

→ When charging, work is done by the battery to ~~move~~ move charge on to the capacitor.

→ Energy is transferred from the power supply & stored as Electrical P.E. in the capacitor.

→ $Q \propto V$ & $\therefore Q = CV$ where C is the constant of proportionality

→ W (work done) (and E_p) = QV

$\therefore \Delta E_p = V_0 \Delta q$ = area under V-Charge graph.

* The energy transferred from the battery when a capacitor is charged is given by the area under the graph line when charge (x-axis) is plotted against potential difference (y-axis).
* Because the graph is a straight line through the origin, $E_p = \frac{1}{2} QV$

$$\boxed{E_p = \frac{1}{2} QV = \frac{1}{2} CV^2 = \frac{Q^2}{2C}}$$

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* Factors affecting capacitance

1. Dielectric material
2. Area of plates (A)
3. ~~Dis~~ Separation of plates (d)

→ $C \propto \frac{A}{d}$

→ For a capacitor with air or a vacuum between the plates, the constant of proportionality is the permitivity of free space.

$\therefore \boxed{C = \frac{\varepsilon_0 A}{d}}$

* Since the use of a dielectric ↑ the capacitance, we introduce a quantity called the "relative permitivity" ε_r of a dielectric.

"The relative permitivity is defined as the capacitance of a parallel-plate capacitor with the dielectric between the plates divided by the capacitance of the same capacitor with a vacuum between the plates". $= \frac{C_{di}}{C_{va}} = \varepsilon_r$

$$\therefore \quad C = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

* Capacitors in series & parallel

Parallel: (~~[illegible]~~ resistors opposite):

$C_{total} = C_1 + C_2 + C_3 \ldots$

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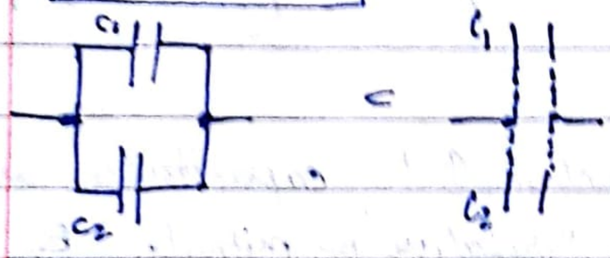
→ The combined capacitance is the sum of their individual capacitances.

→ This is because, when 2 capacitors are connected in parallel together, they are equivalent to a single capacitor with larger plates. ↑ size of plates → ↑ area for charge build-up ⇒ ↑ capacitance.

→ The total charge Q stored by 2 capacitors connected in parallel & charged to a potential difference V is given by:

$\boxed{Q = C_{total} \times V}$



→ Derivation

- Since in parallel, V is the same:

  $Q_1 = C_1 V$ & $Q_2 = C_2 V$

- Total charge stored = sum of these.

  $Q = Q_1 + Q_2 = C_1 V + C_2 V$ → conservation of charge!

- Since V is a common factor

  $Q = (C_1 + C_2)V$

- Comparing this with $Q = C_{total} V$ gives the required

  $C_{total} = C_1 + C_2$

OR $Q = CV = (C_1 + C_2)V$

$\frac{C\cancel{V}}{\cancel{V}} = C_1 + C_2$ ∴ $\boxed{C = C_1 + C_2}$

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SERIES

→ Combined capacitance $= \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \ldots$

→ $V = \frac{Q}{C}$ & in series $V = V_1 + V_2$ addition of pds.

Derivation

$\therefore \frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2}$ where each capacitor has charge Q as current is same in series.

~~Also~~: ~~conservation of charge in series~~.

$\frac{Q}{C} = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$

$\frac{\cancel{Q}}{C} \times \frac{1}{\cancel{Q}} = \frac{1}{C_1} + \frac{1}{C_2} \quad \therefore \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$

→ The reciprocal of the combined capacitance equals the sum of the reciprocals of the individual capacitances in series.

→ For 2 identical capacitors in series, the combined capacitances is equal to half of the value of each one.

→ For capacitors in series, the combined capacitance is ALWAYS LESS than the value of the SMALLEST individual capacitance.

* Capacitance of Isolated bodies:

→ A conducting sphere of radius 'r' insulated from its surroundings & carrying a charge 'Q' will have a potential at its surface ~~of A~~ of 'V' where

$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$ $\quad V = \frac{kQ}{r}$ $\quad F = \frac{kQ_1Q_2}{r^2}$

$F = \frac{qV}{d}$ $= F = \frac{QV}{r} = \frac{kQ_1Q_2}{r^2}$

$V = \frac{k\cancel{Q_1}Q_2\cancel{r}}{\cancel{Q}\cancel{r^2}}$

$\therefore V = \frac{kQ}{r}$

Since $C = \frac{Q}{V}$, it follows that the capacitance of a sphere is

$C = 4\pi\epsilon_0 r$ because $C = \frac{Q}{V} = \frac{\cancel{Q}\times r}{k\cancel{Q}} = \frac{r}{1/4\pi\epsilon_0}$

$= \frac{r}{\cancel{4\pi\epsilon_0}} \times 4\pi\epsilon_0$

$\therefore \boxed{C = 4\pi\epsilon_0 r}$

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