FIELDS OF FORCE

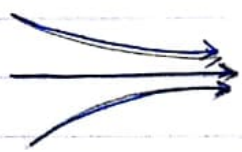
* A field of force is said to exist in a region of space when ~~as~~ suitable object placed in the region experiences a force.
* 3 types of force fields:

1. Gravitational field – in which the object is a mass that experiences a force
2. Electrical field – in which the object is a charge that experiences a force
3. Magnetic field – in which the object is a moving charge or a magnet that experiences the force.

* A field is a vector quantity and has both a magnitude & direction. Represented by drawing lines of force:
  - density of lines indicated magnitude of the field.
  - direction of line, indicated by arrows, shows direction of force on object.
* Field strength (magnitude of field) is ~~obj~~ obtained by measuring the force on the object. The object must be small enough to have no effect on the field being measured.
* Uniform fields: the force is constant at all points

* Non-uniform fields: lines of force change in density as strength of the field changes.

↓ field strength



↑ field strength

* If an object is moved in a field, then energy is either released or supplied, depending on the direction of motion.

→ Object moves against field:
  - energy supplied
  - work done on object
  - increase in potential energy of object.

→ Object moves along field:
  - energy released
  - work done by object
  - decrease in potential energy of object.

$\Delta PE \nearrow$

* It is possible to assign an absolute value to the potential at a point in the field, provided a zero is chosen for the potential. This is usually taken as being when the object is at infinity. The absolute potential at a point in the field is then the work done is moving the object from infinity to the point. & can be +ve ~~OR~~ –ve depending on the field.

Eg: It is convenient to choose an arbitrary zero. In the case of motion under gravity on the Earth, where it is often convenient to choose the Earth's surface as an arbitrary zero.

* Since energy changes as objects move along field lines, lines of equal potential (energy), known as <u>equipotential lines</u>, are always at right angles to the field lines.

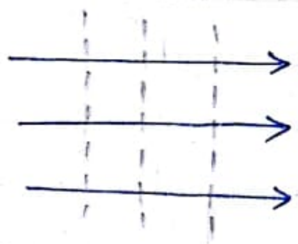
<u>Reason</u>: $\Delta P.E. = \Delta W = F\cos\theta \times s$

When $\theta = 90^\circ$, $F\cos\theta = 0$.

$\therefore$ all points along on the equipotential line have the same potential energy. ~~Moving~~ No work is done in moving an object along this line.

<u>Worked example</u>: Sketch equipotential lines for the following:

1



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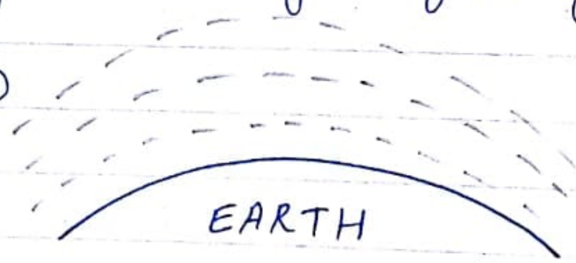

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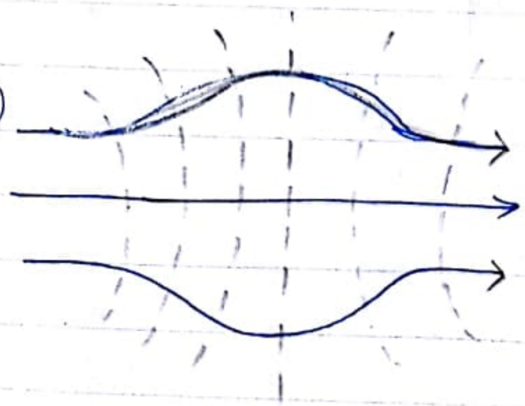


CIRCULAR MOTION

* Newton's first law says that an object continues to move in a straight line unless a resultant force acts on it.
  ∴ To make an object move in a circle you always need a resultant force towards the centre of the circle. This is called a <u>centripetal</u> <u>force</u>. (It is at right angles to the velocity).
* This force, however, does <u>no</u> <u>work</u> on the object because:

(i) At any point the object is moving in a direction along a tangent to the circle. ∴ The force is always ⊥ to the motion. Since $W = F\cos\theta \times s$, $\cos 90° = 0$ ∴ no work done.

(ii) The object does not move in the direction of the force i.e. toward the centre of the circle, so no work done. ($s = 0$)

* If the force is released, the object will fly off tangentially in a straight line.
* The centripetal force is not some new force acting on the object but it is the name given to the resultant force acting on the object in a direction toward the centre of the circle.
* The force can be gravitational, electrostatic, frictional, tension in a string, contact force, etc.
* Angles are measured in <u>radians</u> in circular motion.
* One radian is the angle subtended at the centre of the circle by an arc of length equal to the radius of the circle.

$$\boxed{\text{Angle (in radians) [rad]} = \frac{\text{length of arc}}{\text{radius}} \quad \text{OR} \quad \theta = \frac{s}{r}}$$

Arc length (S) $= 2\pi r \times \frac{\theta}{360°}$ but $360° = 2\pi$ rads

∴ arc length $= 2\pi r \times \frac{\theta}{2\pi} = \theta \times r$ ∴ $\boxed{s = r\theta}$

Angle θ is simply a ratio. It is a dimensionless quantity.

∴ $\theta = \frac{2s}{2r} = \frac{s}{r}$

s = <u>angular displacement</u> = <u>arc length</u>

* Angular displacement for a complete circle:

$$\theta = \frac{\text{circumference}}{\text{radius}} = \frac{2\pi \cancel{r}}{\cancel{r}} = 2\pi = 360^\circ$$

$\therefore$ $180^\circ = \pi$ rad $\qquad \boxed{360^\circ = 2\pi \text{ rad}}$

$90^\circ = \frac{\pi}{2}$ rad

$\boxed{1 \text{ rad} = \frac{360^\circ}{2\pi} = 57.3^\circ}$

$45^\circ = \frac{\pi}{4}$ rad.

* For circular motion, the time period T is the time for one complete rotation. (s)
The frequency, f is the number of rotations per second.

$f = \frac{\text{no. of rotations}}{\text{time taken}}$ $\qquad$ One rotation per sec = 1 Hertz.

* Angular speed $\omega = \frac{\text{angle turned through } (\theta) \text{ [rad]}}{\text{time taken } (t) \text{ [sec]}}$

$\boxed{\omega = \frac{\theta}{t}}$ and is measured in rad s^{-1}

aka angular velocity or angular frequency.
One revolution is 2π radians. This takes T seconds.

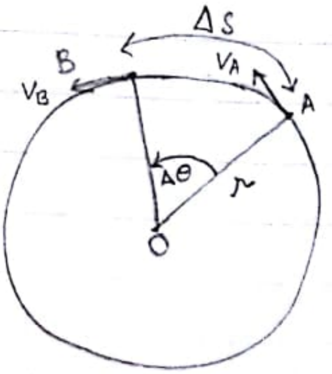
$\omega = \frac{\theta}{T} = \frac{2\pi}{T}$ but $\frac{1}{T} = f$ $\therefore$ $\boxed{\omega = 2\pi f}$ $\qquad \boxed{\omega = \frac{2\pi}{T}}$

* Linear velocity, v, for one revolution:

$v = \frac{\text{distance}}{\text{time}} = \frac{\text{circumference}}{\text{time period}} = \frac{2\pi r}{T}$

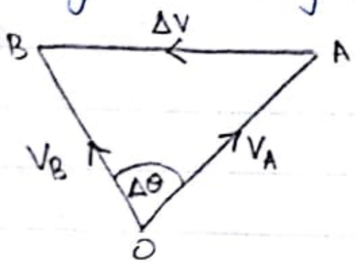
But $\frac{2\pi}{T} = \omega$ so $\boxed{v = r\omega}$

{$v \propto r$}



* Centripetal acceleration.
object travelling at const. speed v from A to B. In time Δt
V_A & V_B are vectors.

The change in velocity Δv may be seen as:



~~ΔV + VA ≠ VB~~ Δv must be added to V_A to give new velocity V_B.
Consider $\Delta\theta$ to be so small that arc AB may be approximated to a straight line.

$\therefore \dfrac{\Delta v}{V_A} = \dfrac{\Delta s}{r}$ (using similar triangles concept).

$\therefore \Delta v = \Delta s \times \dfrac{V_A}{r}$ dividing both equations by Δt:

$\dfrac{\Delta v}{\Delta t} = \dfrac{\Delta s}{\Delta t} \times \dfrac{V_A}{r}$ acceleration $= \dfrac{\Delta v}{\Delta t}$

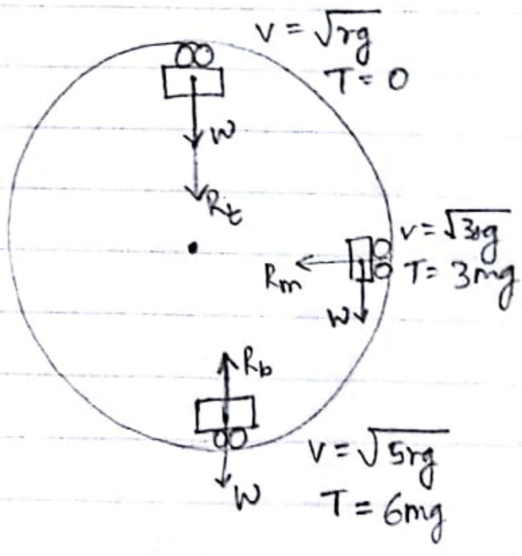
$\therefore a = v \times \dfrac{v}{r} = \dfrac{v^2}{r}$ speed, $v = \dfrac{\Delta s}{\Delta t} = V_A = V_B$

<u>So:</u>

$$\left.\begin{aligned} &① \text{ acceleration} = \frac{v^2}{r} = \underline{r\omega^2} \\ &② \text{ force} = ma = \frac{m \times v^2}{r} = \frac{mv^2}{r} = \underline{mr\omega^2} \end{aligned}\right\} \underline{\text{centripetal}}$$

(force ↓ Newton's 2nd Law.)

* Motion in a verticle circle:



Normal reaction force acts toward centre of the circle, thus providing the required centripetal force.

<u>Bottom</u>: $W - R_b = \dfrac{mv^2}{r}$

Middle: $R_m = \dfrac{mv^2}{r}$

<u>Top</u>: $W + R_t = \dfrac{mv^2}{r}$

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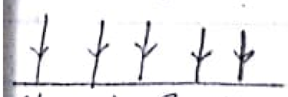
GRAVITATIONAL FIELDS

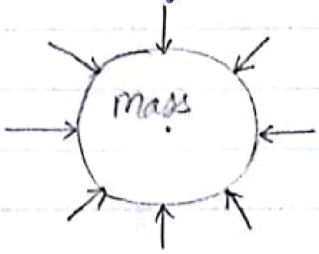
* A gravitational field at a point is detected by observing the force on a mass placed at that point.
* Gravitational fields have the following characteristics:
  1. generated by objects that have mass
  2. always attractive
  3. weakest of the 3 fields.
* Gravitational field strength at a point is defined as the force per unit mass acting on a small mass placed at that point.

  Symbol : g Units : Nkg^{-1} $\therefore\ g = \frac{\text{gravitational force}}{\text{mass}} = \frac{F}{m}$

almost uniform...



Close to Earth

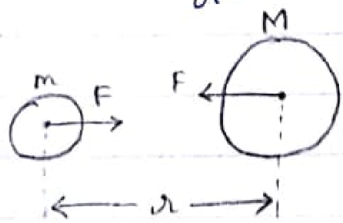


radially inwards lines of force.
At Earth's surface, $g = 9.81\ Nkg^{-1}$
= acc. due to gravity, g, also $9.81\ ms^{-2}$

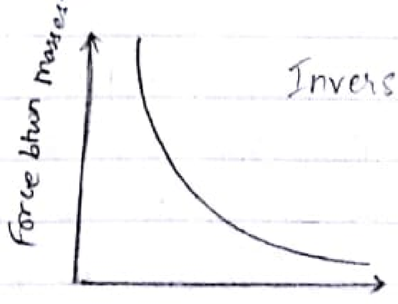
* Newton's Law of Gravitation states:

  Two point masses attract each other with a force that is directly proportional to the product of their masses and inversely proportional to the square of their separation.

  $\therefore\ F \propto \frac{Mm}{r^2}$ OR $F = \frac{GMm}{r^2}$ where G = gravitational constant.
  $= \underline{\underline{6.67 \times 10^{-11}}}\ Nm^2 kg^{-2}$



• follows the inverse square law!

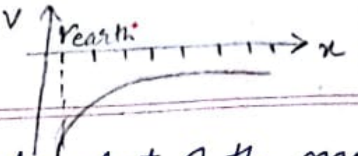


Inverse Square Law Graph.

* To calculate 'g' in a radial field:

  $F = mg$ and $F = \frac{GMm}{r^2}$ $\therefore\ \cancel{m}g = \frac{GM\cancel{m}}{r^2}$ $\therefore\ g = \frac{GM}{r^2}$

(8)



potential gradient

OR acc. of freefall

△ The acceleration due to gravity is independent of the mass 'm' so all objects on the surface of the Earth accelerate at the same rate towards the surface.

* Gravitational potential at a point in a field is defined as the work done in bringing unit mass from infinity to the point.

Symbol: ϕ Units: Jkg^{-1}

$\phi = -\frac{GM}{r}$ The attractive gravitational force between the field-producing mass and test mass gives a -ve potential ∴ (−) sign.

Another symbol: V

work done = ↑ in potential = ΔV = force × distance = $-g \times \Delta x$

$g = -\frac{\Delta V}{\Delta x}$ = potential gradient. x = dist. from Earth's cent

Gravitational field strength is numerically equal to the gravitational potential gradient.

* Minus sign because gravitational field acts in the opposite direction to the distance moved. Work done on object against the field.

✓ Also zero of potential is at $R = \infty$ so potential is -ve as we move closer to mass M.

Energy change (J) = Δ gravitational potential (Jkg^{-1}) × mass (kg)

SATELLITES & ORBITS

* A satellite is any object held in orbit around a larger one by gravitational attraction.
* 4 types of orbit: ① Low-Earth orbit – International Space Station
  ② Polar orbit – TIROS weather satellites
  ③ Eccentric orbit – non-circular path
  ④ Geostationary / geosynchronous orbit. – INTESAT & METEOSAT
* A centripetal force is needed to keep the satellite moving along a circular path.

$F = \frac{GMm}{r^2}$ and $F = \frac{mv^2}{r}$

It is the ~~force that provides~~ the centripetal force as the planet moves in its orbit. 1

$\therefore \frac{GMm}{r^2} = m \times \frac{v^2}{r}$ [Gravitational force of attraction = mass x centripetal acc.]

$F = mg$. Substituting $g = \frac{GM}{r^2}$ gives: g=grav. field strength.

$$\cancel{m}g = \frac{\cancel{m}v^2}{r} \quad \therefore \boxed{g = \frac{v^2}{r}} \text{ or } \boxed{v = \sqrt{rg}}$$ orbital velocity ~~orbital velocity~~

* $F_{grav} = F_{circ}$

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$\frac{GMm}{r^2} = \frac{m}{r} \times \frac{4\pi^2 r^2}{T^2}$$

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

$$\frac{T^2}{r^3} = \frac{4\pi^2}{GM} = \text{constant}$$

$$\therefore \boxed{T^2 \propto r^3}$$

The period T of the planet in its orbit is the time required for the planet travel a distance $2\pi r$. It is moving at speed 'v'. $\therefore v = \frac{2\pi r}{T}$

← Put this here

"For planets or satellites ~~that~~ ~~show~~ describing circular ~~or~~ orbits around the same central body, the square of the period is proportional to the cube of the radius of the orbit."

– Kepler's Third Law of Planetary Motion.

Johannes Kepler analysed data collected by Tycho Brahe.

Later proved by Newton.

Simpler to derive for circular instead of elliptical (reality).

* Geostationary or Geosynchronous orbit: satellites are in an equitorial orbit with exactly the same period of rotation of the Earth, and moves in the same direction as the Earth, so that they are always above the same point on the equator.

$\therefore$ no relative motion ⇒ appears stationary.

→ This idea was suggested by Arthur Clarke & so it is aka "Clarke Belt".

→ Telecommunication ~~and~~ satellites and those responsible for Satellite transmission TV are 'parked' here.

→ Distance from the Earth: using $T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$

$G = 6.67 \times 10^{-11}$ Nm² kg⁻²

$T = 24$ hrs $= 86{,}400$ s

$M = 6.0 \times 10^{24}$ kg

$$\therefore r^3 = \frac{GMT^2}{4\pi^2} = \frac{6.67\times10^{-11} \times 6\times10^{24} \times (86400)^2}{4\pi^2}$$

$$\therefore r = 4.23 \times 10^7 \text{ m (from centre)}$$

(42,300 km) $4.23 \times 10^7 - 6400$ (R of Earth) = 35,900 km (from Earth's surface).
(R+h)

$-ve$ TE $\Rightarrow$ satellite in 'BOUN[illegible]
$+ve/0$ TE $\Rightarrow$ UNBOUND STATE.

* Energy of satellites:

$\rightarrow$ A satellite mass 'm' in an orbit at a radius R has a potential energy

$$E_p = -\frac{GM_E m}{R}$$

$\rightarrow$ The kinetic energy of the satellite $= \frac{1}{2}mv^2$ but $v^2 = \frac{GM_E}{R}$ so:

$$E_K = \frac{1}{2} \times \frac{GM_E m}{R} = \frac{GM_E m}{2R}$$

$\boxed{\text{as } R\downarrow,\ E_K\uparrow \text{ \& orbital velocity } \uparrow.}$

$\rightarrow$ Total Energy $= PE + KE = -\frac{GM_E m}{R} + \frac{GM_E m}{2R} = \boxed{-\frac{GM_E m}{2R}}$

As the satellite moves closer to the Earth, it loses potential energy. Half goes to $\uparrow E_K$ and the other half is lost in friction.
Also, if Earth's surface is considered as zero potential, then $E_p\downarrow$ because height from surface $\downarrow$. $\therefore E_p =$ more $-ve$. (if infinity $= 0$)

* Escape velocity:

$\rightarrow$ To escape from the Earth's, the initial E_K of the satellite must be greater than the gain in E_p as the satellite moves from the surface to an infinite distance away.

$\rightarrow$ gain in $E_p = \underbrace{0}_{E_p \text{ at } \infty} - \Big(\underbrace{-\frac{GM_E m}{R}}_{E_p \text{ on the surface}}\Big) = +\frac{GM_E m}{R}$

$\rightarrow$ initial $E_K = \frac{1}{2}mv^2 =$ gain in $E_p = \frac{GM_E m}{R}$. $v^2 = \frac{2GM_E}{R}$

$\therefore$ escape velocity $v = \sqrt{\frac{2GM_E}{R}} = \sqrt{2gR}$

The orbital velocity of the satellite is $v = \sqrt{\frac{GM_E}{R}}$ but $g = \frac{GM_E}{R^2}$ so

$v = \sqrt{gR}$ so the escape velocity is $\sqrt{2}$ times the orbital velocity

41.2% of V_0 to get V_e. [% $\uparrow$ in vel.]

Q) g, on the moon, is $1.62\ Nkg^{-1}$

$V_e = \sqrt{2}V_0 \Rightarrow \frac{\sqrt{2}V_0 - V_0}{V_0} \times 100 = \frac{1.412 - 1}{1} \times 100 = 41.2\%$

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1. Orbital Velocity : $F_g = F_c$
2. Escape Velocity : $T.E. = K.E + P.E$

$$0 = \frac{1}{2} m v_e^2 + \left(-\frac{GMm}{r}\right)$$

3. Interstellar speed : $T.E.\ (\text{bound}) = T.E.\ (\text{unbound})$

$$\frac{1}{2} m v_o^2 + \left(-\frac{GMm}{r}\right) = \frac{1}{2} m v'^2 + 0$$

$$v_e = \sqrt{\frac{2GM}{r}} \quad \therefore\ v_e^2 = \frac{2GM}{r}$$

$$\frac{v_e^2}{2} = \frac{GM}{r}$$

$$\cancel{m}\left(\frac{1}{2} v_o^2 - \frac{GM}{r}\right) = \cancel{m}\left(\frac{1}{2} v'^2\right)$$

$$\frac{v_o^2}{2} - \frac{v_e^2}{2} = \frac{v'^2}{2}$$

$$\therefore\ v'^2 = v_o^2 - v_e^2$$

$$v' = \sqrt{v_o^2 - v_e^2}$$

v = velocity of projection from SURFACE.

* Effect of depth & height on 'g':

$$g_d = g_s\left(1 - \frac{d}{r}\right)$$ where d = depth below surface, r = radius of Earth.

$$g_h = g_s\left(1 - \frac{2h}{r}\right)$$ where h = height above surface

&

$$g_h = g_s\left(\frac{R}{R+h}\right)^2$$