

IDEAL GASES

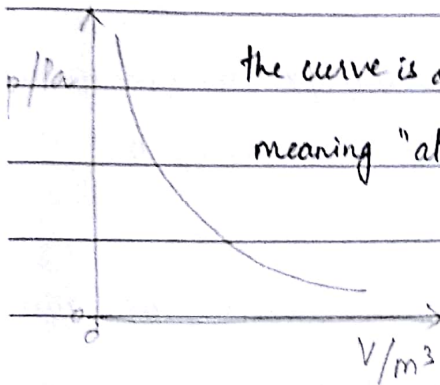
* Boyle's law - "The pressure exerted by a fixed mass of gas is inversely proportional to its volume, provided the temperature of the gas remains constant."

- discovered by Robert Boyle in 1662.

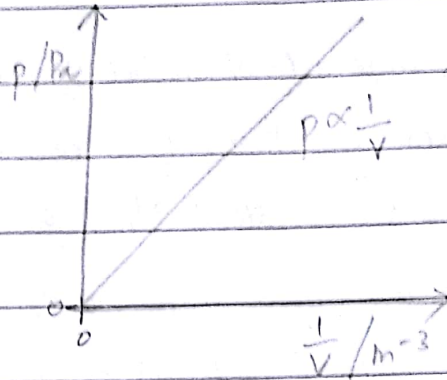
- $P \propto \frac{1}{V}$ @ const. temp & mass. $\therefore p \propto \frac{1}{V}$

- $pV = \text{constant}$

- $p_1 V_1 = p_2 V_2$



the curve is an "isotherm" meaning "at same temp."



$p \propto \frac{1}{V}$

* Work done on gas during compression = Force \times distance

but distance = $\frac{\Delta \text{volume}}{\text{area}}$ \therefore Work done (ΔW) = pressure (p) \times ΔV

[Joules] [Pascals] [cubicmetre]

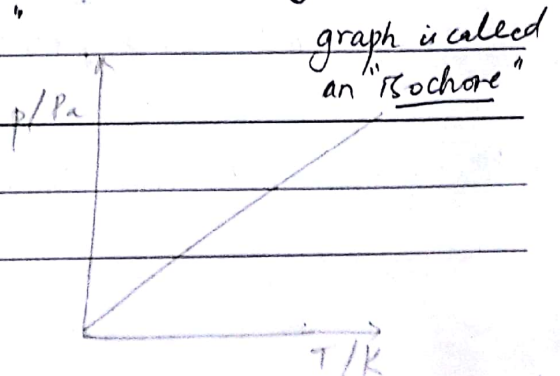
* Gay-Lussac/Amonton's law - "The pressure exerted by a fixed mass of gas is directly proportional to its temperature, provided the volume of the gas remains constant."

- $P \propto T$ @ constant volume

- $\frac{P}{T} = \text{constant}$

- $\frac{P_1}{T_1} = \frac{P_2}{T_2}$

T_1 & T_2 are the absolute temps. before & after the change



HOMEWORK

* Charles's Law - "The volume of a fixed mass of gas is directly proportional to its temperature, ^{so that} provided the pressure remains constant."

$$- V \propto T$$

- graph would be called an "isobar"

$$- \frac{V}{T} = \text{constant}$$

$$- \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

* Ideal gases would follow/obey these laws exactly. But at high pressures or low temperatures, real gases behave differently.

Under these conditions, a real gas could condense into a liquid!

* Definitions:

1. Mole: the amount of matter that contains as many particles as there are atoms in 12g of isotope Carbon-12. This number is the Avogadro constant $N_A = 6.022 \times 10^{23} \text{ mol}^{-1}$

The mole is a measure of the amount of a substance, ~~as~~ specified in terms of a fixed number of atoms/molecules.

2. Pressure: the perpendicular force a gas exerts per unit area. (Nm^{-2} or Pa)

3. S.t.p: standard atmospheric pressure & temperature - 1 atm or $1.01 \times 10^5 \text{ Pa}$ & 0°C or 273.15 K

* Ideal Gas Equation:

Putting all the laws together, we get the relation: $pV \propto mT$

$$pV \propto AmT \quad A = \text{constant of proportionality}$$

but $m = n \times M$ \therefore mass of 1mol of element is its nucleon number in grams.

$$\therefore pV \propto nT \Rightarrow \boxed{pV = nRT} \text{ where } R = \text{universal gas constant} \\ = 8.31 \text{ JK}^{-1} \text{ mol}^{-1}$$

Aka: $pV_m = RT$ where $V_m = \text{molar volume} = \text{volume occupied by one mole of gas}$

⊛ An ideal gas is one which obeys the equation of state $pV = nRT$ at all pressures, volumes and temperatures.

KINETIC THEORY OF GASES

→ It is a theory which links the microscopic properties of particles to the macroscopic properties of a gas.

→ It considers a gas as molecules in a random motion, and states that the pressure of a gas on the walls of a container is due to the collision of gas molecules with the walls.

→ Assumptions:

1. A gas consists of a large number of molecules (spherical particles)
2. The molecules are in rapid, random motion
3. Collisions between molecules are elastic
4. There are no attractive forces between molecules
5. The molecules are small compared with the volume of the container so their own volume is negligible.
6. When the molecules collide, there are repulsive forces, but the time of the collision is negligible compared with the time between collisions.

* Pressure of an ideal gas = $\frac{1}{3} \rho \bar{c}^2$ $\rho = \text{density of gas}$
 where \bar{c}^2 (mean-square speed) = $\frac{c_1^2 + c_2^2 + c_3^2 + \dots}{N}$

* Internal energy of an ideal gas: (Only E_k as we are disregarding attractive forces).
 $U = \frac{3}{2} nRT$

* Kinetic energy of one molecule:

$$E_k \text{ of 1 mole of ideal gas} = \frac{3}{2} nRT \quad n=1$$

$$\text{but 1 mole} = N_A \text{ molecules} \therefore E_k \text{ of one molecule} = \frac{3}{2} \frac{RT}{N_A} = \frac{3}{2} kT$$

$$k = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

k = Boltzmann constant \Rightarrow gas constant for one molecule.

$$k = \frac{R}{N_A} \quad \therefore \boxed{E_k = \frac{3}{2} kT}$$

* Pressure exerted by one molecule: (\ddot{x})

$$t = \frac{L}{c_x} \text{ (dist)} \\ c_x \text{ (speed)}$$

$$\Delta \text{ momentum on collision} = 2mc_x$$

$$\rightarrow + \leftarrow = 2t$$

$$\text{Force} = \text{rate of change of momentum} = \frac{\Delta p}{\Delta t} = \frac{2mc_x}{2L/c_x} = \frac{mc_x^2}{L}$$

$$\text{If there are } N \text{ molecules, avg. force} = \frac{Nmc_x^2}{L}$$

$$\text{pressure exerted} = \frac{\text{force}}{\text{area}} = \frac{Nmc_x^2}{L} \cdot \frac{1}{L^2} = \frac{Nmc_x^2}{L^3}$$

But Volume of container = $L^3 \therefore \text{Work done} = \text{pressure} \times \text{volume}$

$$\therefore \text{pressure} = \frac{Nmc_x^2}{V} = \frac{Nmc_x^2}{L^3} \times L^3$$

This is considering velocity only in the 'x' direction, taking avg gives

$$c_m^2 = \frac{c_x^2 + c_y^2 + c_z^2}{3}$$

$$\therefore \boxed{pV = \frac{1}{3} Nm \langle c^2 \rangle} \quad \text{OR} \quad \boxed{p = \frac{1}{3} \frac{Nm}{V} \langle c^2 \rangle}$$

$$\frac{Nm}{V} = \rho \quad \text{so} \quad \boxed{p = \frac{1}{3} \rho \langle c^2 \rangle}$$

Linking the macroscopic properties of gases to microscopic props. of gas molecules.

$$pV = nRT = \frac{1}{3} Nm \langle c^2 \rangle$$

$$\frac{1}{3} Nm \langle c^2 \rangle = nRT, \text{ (trying to relate temp. (macro) to mass \& speed}$$

$$m \langle c^2 \rangle = \frac{3nRT}{N} \text{ of the molecules.}$$

$$\frac{N}{n} \frac{M}{M} = N_A \quad \text{So:} \quad m \langle c^2 \rangle = \frac{3RT}{N_A}$$

$$\text{But } \frac{R}{N_A} = k \text{ (Boltzmann Constant)} \quad \therefore m \langle c^2 \rangle = 3kT$$

dividing both sides by 2:

$$(KE) \quad \frac{1}{2} m \langle c^2 \rangle = \frac{3}{2} kT$$

Hence the thermodynamic temp T is proportional to avg. KE of a molecule.

"The mean translational KE of an atom/molecule of an ideal gas is proportional to the thermodynamic temp"

translational \rightarrow moving along; rotational \rightarrow spinning/rotating

$$\therefore \langle c^2 \rangle = \frac{3kT}{m} \quad \text{and} \quad \sqrt{\langle c^2 \rangle} = \sqrt{3kT/m}$$

$\sqrt{\langle c^2 \rangle}$ is called root-mean-square speed or r.m.s speed (c_{rms})

of the molecules. It is not always ~~the~~ equal to the avg KE of molecules

but considered so ~~at it is~~ as avg KE = $0.92 \times c_{rms}$.

$c_{rms} \propto \sqrt{T}$ of gas & $c_{rms} \propto \frac{1}{\sqrt{m}}$ of molecule.

\therefore at a given temp, less massive molecules move faster, on avg, than more massive molecules. \uparrow temp, \uparrow ~~temp~~ speed.

$T \propto \text{k.e.} \propto v^2$ \therefore doubling $T \Rightarrow$ doubled KE $\Rightarrow \uparrow v$ by $\sqrt{2}$

All molecules in a gas have same avg translational K.E. \therefore

\uparrow mass = \downarrow velocity.