OSCILLATIONS

→ Oscillation occurs when an object undergoes a displacement repeatedly on either side of an equilibrium position. If the motion is repeated after a fixed period of time, it is called "periodic motion".

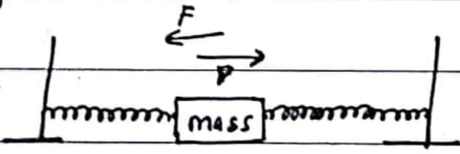
E.g. vibrating strings, the pendulum on a clock, a mass vibrating on the end of a spring, car suspension systems, loudspeakers, etc.

→ A particle is said to be undergoing free oscillations when the only external force acting on it is the restoring force.
There are no forces to dissipate energy & so the oscillations have constant amplitude. Total energy remains constant. Simple harmonic oscillations are free oscillations.

→ Oscillators that have a constant time period are called isochronous & are made use of timing devices — quartz.

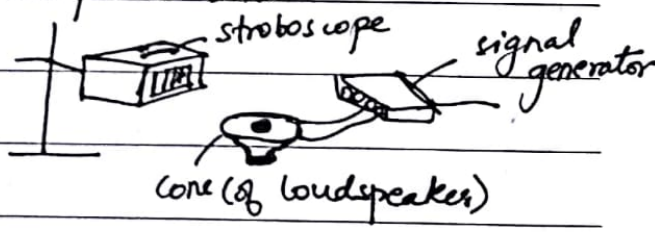
→ Investigating motion of an oscillator:

① A mass – spring system:

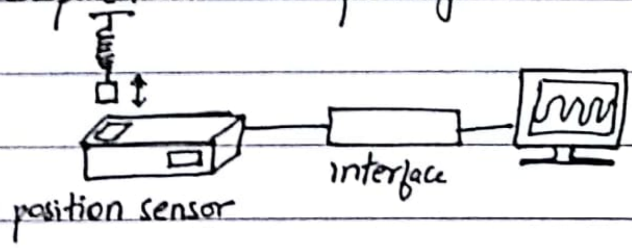


② A long pendulum:

③ A loudspeaker cone:



④ Displacement – time plotting.



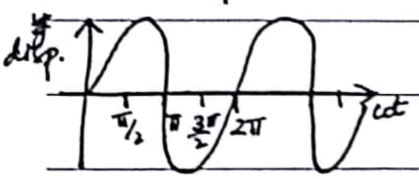
* A sinusoidal (sine or cosine) d-t graph is characteristic of s.h.m.

* The amplitude (a scalar quantity) is the maximum displacement.
* The distance of the oscillating mass from the equilibrium position is known as the displacement. ~~It can~~ [The disp. can be any physical parameter that varies with time: particle position, electric field etc. ⇒ ref. to d-t graphs].
* The time taken for one complete oscillation is referred to as the period T of the oscillation. $\boxed{f = 1/T \text{ or } T = 1/f}$
* The number of oscillations per unit time is the frequency f.

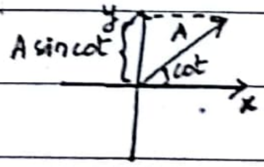
→ Harmonic oscillations are those whose displacements as a function of time vary according to sine or cosine functions. It can be constructed by considering a rotating arm, called a phasor, of size equal to the amplitude & rotating with constant angular frequency/velocity.

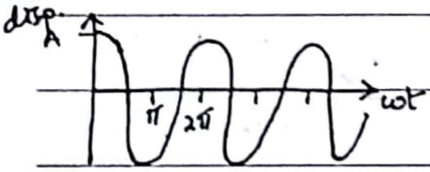
* Angular frequency (ω) is the speed of rotation of the phasor measured in radians per second.

$T = \frac{\text{distance}}{\text{speed}} = \frac{2\pi}{\omega}$ but $T = \frac{1}{f}$ $\therefore \frac{1}{f} = \frac{2\pi}{\omega}$ $\therefore \boxed{\omega = 2\pi f}$

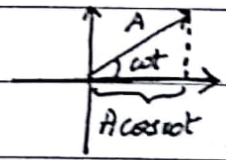


$y = A \sin \omega t$
$= A \sin 2\pi f t$





$y = A \cos \omega t$
$= A \cos 2\pi f t$



→ horizontal axis $= \omega t$ = the angle turned through in radians, ↑ with time.

* Phase difference is the angle ~~by~~ or amount by which one oscillations lags behind another. Can be measured as a fraction of an oscillation. Phase describes the point that an oscillating mass has reached within the complete cycle of an oscillation.

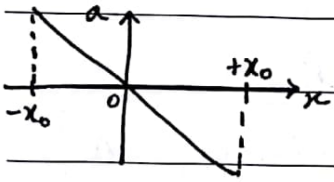
$\frac{t}{T} \times 360°$ or $\frac{t}{T} \times 2\pi$ rad | $\frac{d}{\lambda} \times 360°$ or $\frac{d}{\lambda} \times 2\pi$ rad.

7. Diff between disp/t graph & v/t graph = 90° or $\frac{1}{2}\pi$ rad.

equilibrium position

S.H.M. : Is defined as the motion of a particle/object about a fixed point such that its acceleration 'a' is proportional to its displacement 'x' from the fixed point and is directed towards the point.

→ $a = -\omega^2 x$ (defining equation of s.h.m)



a-x graph for s.h.m

- magnitude of gradient = ω^2
- the gradient is independent of the amplitude of the motion. This means that the frequency for the period of the oscillator is independent of the amplitude & so a simple harmonic oscillator keeps steady time.

→ solution to above equation: $x = x_0 \sin \omega t$ [sine curve] tells how disp varies with time.

$x = x_0 \cos \omega t$ [cosine curve]

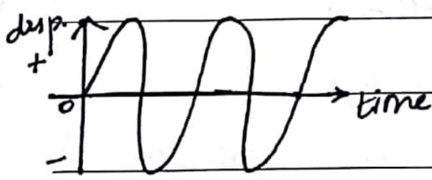
→ $v = v_0 \cos \omega t$ tells how v depends on t.

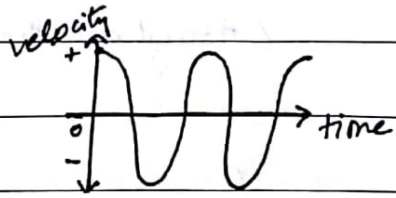
→ $v = \pm \omega \sqrt{(x_0^2 - x^2)}$ tells how v depends on oscillator's disp. x. can deduce speed at any point in an oscillation

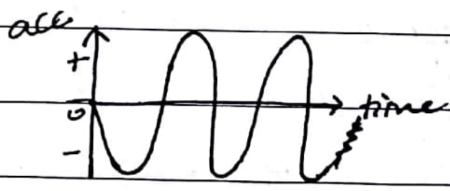
→ $v_0 = \omega x_0 \Rightarrow v_0 \propto x_0 \Rightarrow v_0 \propto \omega$ } if $x = 0$ i.e. at equilibrium position.

$\therefore v_0 = (2\pi f) x_0$

→ $v \propto 1/x$ and $a \propto x$ and $v \propto 1/a$ $\quad a_{max} = -\omega^2 x_0$





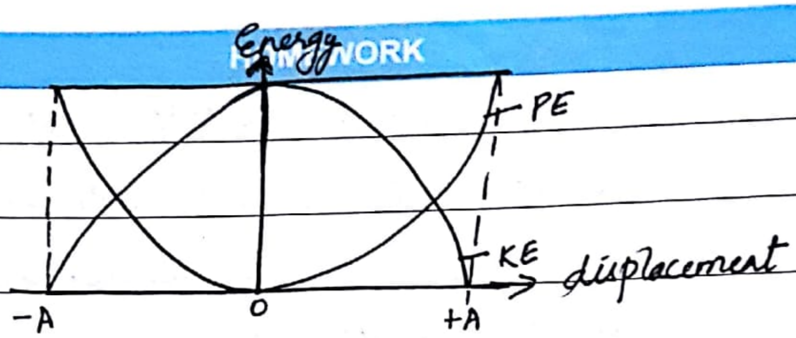


grad = velocity $\qquad$ grad = acceleration

(grad is steepest when disp. is zero. ∴ max vel.)

* acc. graph opposite of x graph.

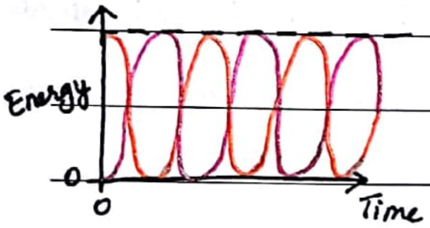
Energy in S.H.M:



↑ energy into system = ↑ oscillation size

@ equilibrium position ⇒ max vel. ⇒ total energy = K.E

$= \frac{1}{2} m v_{max}^2 = \frac{1}{2} m (2\pi f A)^2 = \frac{1}{2} m (4\pi^2 f^2) A^2$ ∴ $\boxed{\text{Total energy} \propto A^2}$

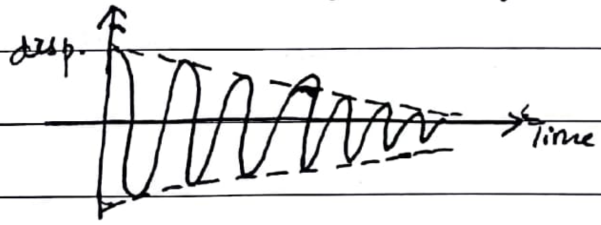


KE & PE of an oscillator vary periodically, but the total energy remains constant if the system is undamped.

$P.E = \underset{\substack{\downarrow \\ T.E.}}{K.E_{max}} - K.E = \frac{1}{2} m\omega^2 A^2 - \frac{1}{2} m\omega^2 (A^2 - x^2) = \boxed{\frac{1}{2} m\omega^2 x^2}$

DECAY OF S.H.M. & DAMPING

* In real systems, the total energy is not constant and energy is lost mainly due to frictional forces.



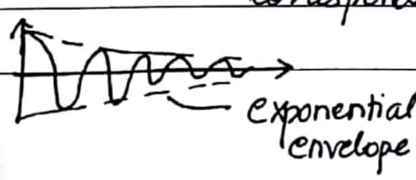
Amplitude of oscillation decays over time & the oscillations are said to be damped. The time period remains the same (throughout the motion)

* Opposite of free oscillations.
* Total energy of the oscillation decreases with time. Dissipated as heat.

a) Light damping – amplitude of oscillation decreases gradually with time

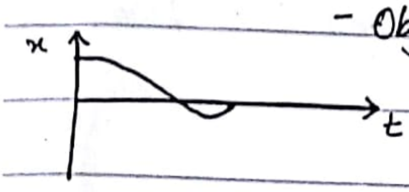
Eg. drums in air. – decrease in amplitude is exponential with time.

– period of oscillation is slightly greater than that of the corresponding free oscillation



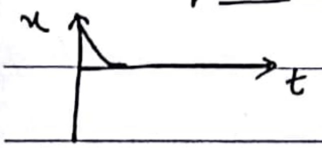
At first, oscillates rapidly ∴ ↑ air resistance
as speed ↓, air resistance ↓ ∴ amplitude ↓ gradually
frequency does NOT change. Characteristic of SHM.

b) Heavier damping – Object takes longer to return to equilibrium position.
– Object hardly oscillates at all.



c) Critical damping – as in car suspension systems to prevent prolonged vibration after going over a bump in the road.
– displacement ↓ to zero, in the shortest time without any oscillation.

° Damping that allows the object to move back to its equilibrium position in the quickest possible time without oscillating is called critical damping.



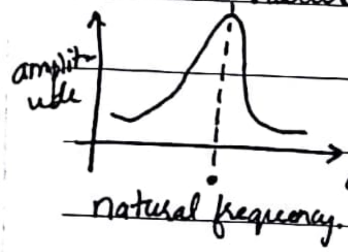
d) Over-damping – damping ↑ over critical level
– no oscillation
– object takes a long time to return to equilibrium position.
E.g. ① a pendulum through thick treacle.
② swing door dampers → ensure door does not overshoot closed position.
③ Useful where rapid fluctuations need to be ignored. E.g. car fuel gauge. Overdamping stops the pointer oscillating as the fuel sloshes about in the tank.

*. Damping is achieved by introducing the force of friction into a mechanical system.

FORCED OSCILLATIONS & RESONANCE

* When a vibrating body undergoes free oscillations, it vibrates at its natural frequency.
* Vibrating objects may have periodic forces acting on them which make the object vibrate at the frequency of the applied force, rather than the natural frequency of the system. The object is then said to be undergoing forced oscillations.

* Resonance occurs when the driving frequency of an object is equal to its natural frequency, giving a maximum ~~of~~ amplitude of vibration



← resonance curve

→ Causes the violent vibrations of a washing machine at some spin speeds.

→ Can be destructive: vibrations can build up to dangerous levels. Example when opera singers hit a high note... frequency of sound matches natural frequency of the glass. The glass then resonates, vibrating more & more until it breaks.

→ Can be useful: • Tuning circuits in TV and radio sets work by resonating at the frequency of the station you set.

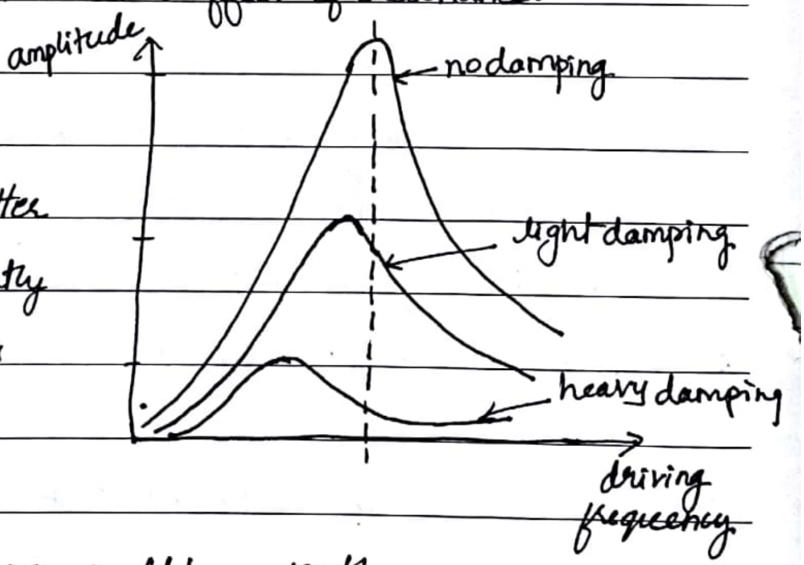
• Digital watches rely on the resonant vibration of a quartz crystal to keep time

• Wind instruments produce sound by forcing the air inside the tube to resonate.

* Damping absorbs energy & reduces the effect of resonance.

→ As damping ↑:

1. amplitude of resonance peak ↓
2. resonance peak gets broader/flatter
3. the 'resonant frequency' gets slightly lower i.e. shifts gradually towards lower frequencies (peak moves left).

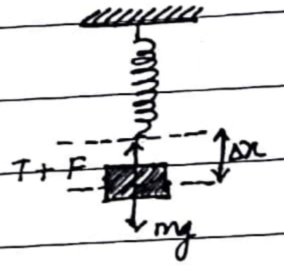


→ Damping is used where resonance could be a problem.

E.g. damping of buildings in earthquake zones. The foundations are designed to absorb energy. This stops the amplitude of the building's oscillations reaching dangerous levels when an earthquake wave arrives.

Examples of SHM.

1. Mass on a helical spring: $F = -k\Delta x$



Restoring force ∝ displacement

Acceleration ∝ displacement ∴ S.H.M.

$T = 2\pi\sqrt{\frac{m}{k}}$ m & k (stiffness) affect T.

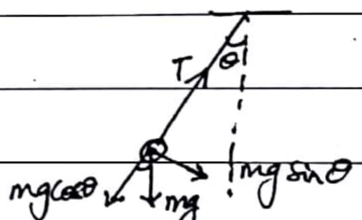
T = ~~tension~~. period of oscillations.

For oscillations to be S.H.M., the spring must obey Hooke's law throughout. i.e. extensions must not exceed limit of proportionality. For large amplitude oscillations, the spring may become slack. Spring must ideally be massless (assumption).

This is particularly useful in modelling the vibrations of molecules.

2. The simple pendulum:

→ A pendulum oscillates with SHM provided the amplitude the amplitude is small



→ Time period NOT affected by mass!

$T = 2\pi\sqrt{\frac{l}{g}}$

→ has angular displacement θ. (in radians).

→ $mg\sin\theta$ is the restoring force ∴ acc. towards equil. pos.

∴ restoring force depends on $\sin\theta$. $\theta \approx \sin\theta$ for small angles.

∴ for angles < 5°, the pendulum bob oscillates with s.h.m.

→ l = distance between centre of mass and point of suspension.

3. Measuring acc. due to gravity, g (related to pendulum)

$T = \frac{2\pi}{\sqrt{g}} \times \sqrt{l}$ ∴ graph of T against $\sqrt{l}$ (OR T^2 against l) gives gradient $2\pi/\sqrt{g}$ (OR $4\pi^2/g$)

of the form:

$y = m \times x + c$ By finding gradient, g can be worked out!

$\therefore m = \frac{2\pi}{\sqrt{g}} = \frac{4\pi^2}{g}$

HOMEWORK

NOTE:- In General SHM.

(a) Restoring force, $F = -ma = -m(\omega^2 y)$

Hooke's law, $F = -Ky$

$\therefore -m\omega^2 y = -Ky \Rightarrow K = m\omega^2 \Rightarrow \omega^2 = \frac{K}{m}$

$\therefore \omega = \sqrt{\frac{K}{m}}$; As $\omega = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\omega}$

(4) $\therefore T = \frac{2\pi}{\sqrt{\frac{K}{m}}} \Rightarrow \boxed{T = 2\pi\sqrt{\frac{m}{K}}}$ (m = inertia factor. K = spring factor.)

(b) Further, as $F = -Ky$, or, $F = Ky$ (ignoring -ve sign, which is only indicative of direction)

In LINEAR SHM

$T = 2\pi\sqrt{\frac{m}{K}}$ $\qquad \Downarrow \quad K = \frac{F}{y}$

$\therefore T = 2\pi\sqrt{\frac{m}{F/y}} = 2\pi\sqrt{\frac{m \cdot y}{F}} = 2\pi\sqrt{\frac{m \cdot y}{m \cdot a}}$ (as $F = m \cdot a$)

$\therefore \boxed{T = 2\pi\sqrt{\frac{y}{a}}}$

(6) $\therefore T = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}}$

NOTE: In linear SHM, inertia factor stands for mass of body executing SHM & spring factor stands for Force per unit displacement.